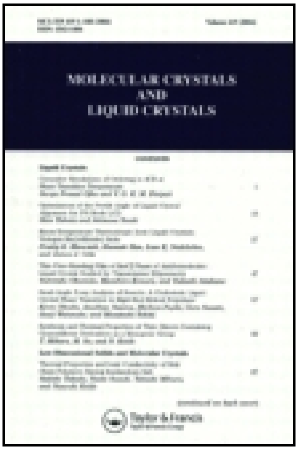


Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

Quantum Stochasticity in the Stokes Parameters of Light, Polarization, Switching and Procedure of Nondemolition Measurements of Distributed Feedback Systems

Alexander P. Alodjants [a], Andrei Yu. Leksin [a] & Sergei M. Arakelian [a]

[a] Department of Physics and Applied Mathematics, Vladimir State University, Valdimir, 600026, Russia

Version of record first published: 24 Sep 2006

Mol. Cryst. Liq. Cryst., Vol. 321, pp. 223-236
Reprints available directly from the publisher
Photocopying permitted by license only

Quantum Stochasticity in the Stokes Parameters of Light, Polarization, Switching and Procedure of Nondemolition Measurements of Distributed Feedback Systems

ALEXANDER P. ALODJANTS, ANDREI YU. LEKSIN, and
SERGEI M. ARAKELIAN
Department of Physics and Applied Mathematics, Vladimir State University,
Valdimir, 600026 Russia

The analysis of formation of chaotic quantum states for two (or four) orthogonally polarized modes in a tunnelly-coupled optical fiber (TCOF) has been carried out. A special attention is devoted both to the structure of arising polarization instabilities and to the possibility to control of such a quantum state by procedure of quantum nondemolition measurements. New effect of polarization switching is obtained. A new method of generating correlated fluctuations in optical field, based on Raman-Nath diffraction by a fine grating, is discussed. A nonclassical effect is predicted: a suppression of quantum fluctuations below the level of full coherence in the sum of difference photon numbers for the transmitted and scattered waves. The physics of this phenomenon involves a correlation of quantum fluctuations in the intensities of two modes that are coupled nonlinearly. This correlation is most effective in the limit of parametric interaction between the waves in the medium.

Keywords: light polarization; switching; quantum nondemolition measurement; correlated fluctuations

INTRODUCTION

Quantum and classical states of light in a nonlinear system with a distributed feedback (DFB) have a special interest at present[1,2]. The high efficiency energy exchange (due to both linear and nonlinear processes for two modes) results in excitation of the wave instablities in such a system as well. Besides that, the switching of intensities for two interacted modes takes place. The study of these two problems for polarization characteristics of light in a special type of optical fibers with non-local interaction is presented in our paper for (i) two traveling polarization modes; (ii) four wave mixing of two waves with orthogonal polarization.

BASIC EQUATIONS AND GENERAL DESCRIPTION

Let us describe the propagation of four orthogonally polarized modes (two modes (a, b) and two components ($\pm$) for each mode) with same frequencies in both a periodically-inhomogenous[6] or a dual core tunnelly-coupled[7,8] optical fibers (TCOF) by the equations for slowly varying complex amplitudes $a_\pm$ and $b_\pm$ as follows

$$da_\pm / dz = \frac{ia_\mp}{2L_B} + \frac{i}{L_{NL}}\left(|a_\pm|^2 + 2|b_\pm|^2\right)a_\pm; \tag{1a}$$

$$db_\pm / dz = \frac{ib_\mp}{2L_B} + \frac{im}{L_{NL}}\left(|b_\pm|^2 + 2|a_\pm|^2\right)b_\pm; \tag{1b}$$

where L_B determines the spatial linear beat length of the energy exchange between two orthogonally polarized modes; $m \equiv \Theta_p/\Theta_s$ where $\Theta_{s,p}$ determines the component of the third-order optical susceptibility of the fiber; the parameter $L_{NL} \cong 1/(\Theta_s I_0)$ characterizes the effective spatial length for nonlinear interaction of two modes (the modes $a_\pm$ and $b_\pm$ are normalized to the the total intensity $I_0 \equiv |a_+|^2 + |a_-|^2 + |b_+|^2 + |b_-|^2$ of the waves at the input of the optical fiber). For the case of two modes interaction[6] (for example, a_+

and a_-) being in a twisted birefringent optical fiber or in a TCOF we can omit the equations (1b) and all the terms with b_+, b_- in eq. (1a).

To describe the interaction of two modes in polarization parameters for the TCOF we introduce the Stokes parameters of light[8] S_{ja} (j=0,1,2,3):

$$S_{0,1} = |a_-|^2 \pm |a_+|^2, \quad S_2 = a_+^* a_- + a_-^* a_+, \quad S_3 = i\left(a_+^* a_- - a_-^* a_+\right) \tag{2}$$

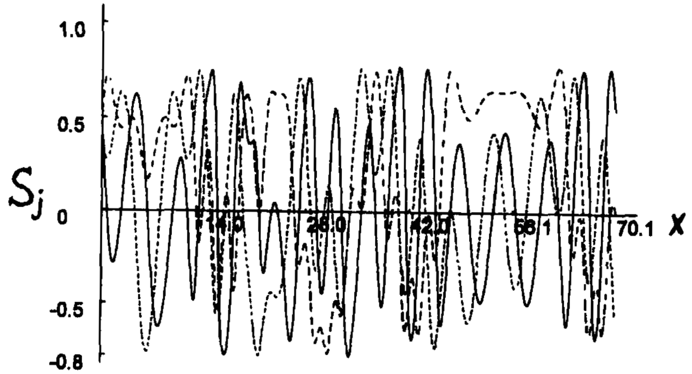
In the case of four wave mixing process in optical fiber an additional set of the Stokes parameters P_j should be taken into account to characterize the total four-mode polarization state of optical field. Geometrically any polarization state of optical field described by the Stokes parameters (2) can be represented as a point on the Poincare sphere in 3D-space of S_1, S_2, S_3 (and/or P_1, P_2, P_3). The measurement of the Stokes parameters reduces to well known techniques based on the combination of photodetectors, wave plates and beam splitters[9].

POLARIZATION INSTABILITIES. NUMERICAL RESULTS

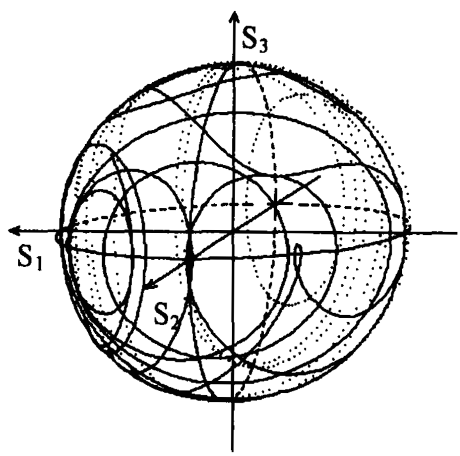
In general case the equations (1) have the following integrals of motion:

$$R_S^2 = S_1^2 + S_2^2 + S_3^2 = S_0^2 \equiv S_{00}^2, \quad R_P^2 = P_1^2 + P_2^2 + P_3^2 = P_0^2 \equiv P_{00}^2,$$
$$\Gamma = S_2 + \frac{1}{m} P_2 + \frac{\beta}{2} S_1^2 + \frac{\beta}{2} P_1^2 + 2\beta S_1 P_1, \tag{3}$$

where $\beta \equiv L_B / L_{NL}$; S_0, P_0 represent the intensities of the polarization modes at the input of the medium (see (2a)). Thus, the first two relations in (3) determine the energy conservation law in the system.



(a)



(b)

FIGURE 1 (a) The dependences for the normalized Stokes parameters S_j (j=1,2,3) for one pair of polarized modes as a function of normalized length $x \equiv z/L_B$ in a TCOF, (b) the corresponding trajectory on the Poincare sphere. (the values of the Stokes parameters at the input of the medium are $S_{10} = S_{20} = S_{30} \approx 0.4472$, $P_{10} \approx 0.2236$, $P_{20} \approx 0.0283$, $P_3 \approx 0.00258$, $m = 0.001$, $\beta = L_B/L_{NL} = 2$. Here and below the dotted curve (……)corresponds to the S_1 Stokes parameter, dashed curve (- - - -) denotes the S_2 Stokes parameter and solid curve (——) corresponds to the S_3 one.)

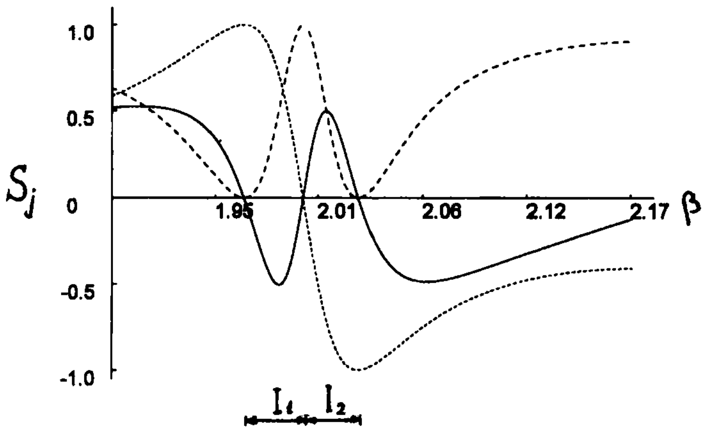


FIGURE 2 The dependences for the normalized Stokes parameters S_j (j=1,2,3) vs the parameter $\beta = L_B/L_{NL}$ (i.e. input intensity) corresponding to the polarization switching region (I_1 and I_2 are the intensities of the switching). The input polarization states are determined by the Stokes parameters $S_{10} = -1$, $S_{20} = S_{30} = 0$ and by the coordinate $x = 2\pi$.

Let us consider the solution of the equations (1a) for the case of two modes only ($P_j = 0$) in the case when $L_B \sim L_{NL}$. The dependences for the Stokes parameters S_j (j=1,2,3) of one polarization mode (2) as a function of $x = z/L_B$ are shown in Fig.1a. The corresponding trajectory in the subspace S_1, S_2, S_3 on the Poincare sphere is displayed in Fig.1b. In this case the behavior of the Stokes parameters P_j is regular. At the same time the behavior of another set of the Stokes parameters S_j is more complicated. It is easy to see from Fig.1b that the polarization state, i.e. the point on the Poincare sphere, roams and so, the regime of polarization instability occurs.

THE SWITCHING EFFECT FOR POLARIZATION STATE OF LIGHT

In this section we consider the new switching phenomena for the light polarization. Although the switching effect for intensity of light in two mode system is very well known in classical optics[7] but real physical interpretation should be more quantum than classical. In fact, the tunneling process for photons redistributed in the fiber from one core to another one can be recognized as a *macroscopic quantum effect*.

The dependences for the Stokes parameters S_j as a function of the $\beta \equiv L_B/L_{NL}$ parameter in the case of two circular polarization mode interaction ($P_j = 0$) are shown in Fig.2 (at the input of the fiber $S_{10} = -1, S_{2,30} = 0$). The points M_0 and M_1 correspond to the two polarization states for output light just before and after the switching process accordingly.

Switching polarization effects could be obtained for the all three Stokes parameters S_j (j=1,2,3) but for the phase-depended parameters S_2 and S_3 the phase properties of the polarization modes control the process as well. A switching intensity δI is determined by variation of the effective parameter β (Fig.2). Let us define a jump parameter ξ as a ratio of two values, i.e. the variation of one of the Stokes parameter δF (being a difference between two values of the S_j-parameter characterizing a switching effect) and the switching intensity δI:

$$\xi = \delta F/\delta I \tag{4}$$

Numerical results for approximately linear input polarization ($S_{10} = 0.1$, $S_{20} \approx 0.9434$, $S_{30} \approx 0.3162$; $x = 2\pi$) are:

$$\begin{aligned} &\xi_1 \approx 7.33 \cdot 10^5 && \text{for the } S_1\text{-parameter switching} \\ &\xi_2 \approx 2.3 \cdot 10^6 && \text{for the } S_2\text{-parameter switching} \\ &\xi_3 \approx 2.56 \cdot 10^6 && \text{for the } S_3\text{-parameter switching} \end{aligned} \tag{5}$$

The final result obtained from (5) is very excited. In fact we can control the output parameter (e.g. S_1) by very small variations of the signal mode intensity at the input of the medium which is approximately in 10^6 times smaller than a pump intensity. But we must take into account the fluctuations of the optical fields in this case and separate analysis should be given.

We also carried out the calculations for two pairs of the modes in each core, i.e. in the case of four mode interaction in a TCOF. In this case we have the input intensity region resulting in strong oscillations (switching) for the output Stokes parameters.

CONCLUSION

Finally, let us briefly discuss the possibility of experimental observation of the effect of polarization switching and instability behavior in the light polarization characteristics. For the last case an experimental verification of considered nonclassical states could be realized by the precision polarization measurements on the basis of quantum nondemolition (QND) measurements for the Stokes parameters of light [10,13]. Such a procedure allows to measure for example the S_{1n} Stokes parameter (in particular for a stochastic behavior of the system) by other parameter (e.g. S_{3n}) using a special technique (the QND-apparatus). For such a case we are able both to localize (select) a quantum state of the system and to suppress the fluctuations for a measure parameter without its demolition. The fact means that a separation of the attractors with stable and unstable quantum states arising in the system becomes possible in an experiment.

APPENDIX. CORRELATION OF QUANTUM INTENSITY FLUCTUATIONS IN RAMAN-NATH DIFFRACTION IN A NLC.

In this division we briefly propose a new method for generating fields that possess the quantum properties. Our method has a number of features which distinguish it from conventional schemes (e.g., schemes that use parametric generation), a fact that is especially relevant to experiment.

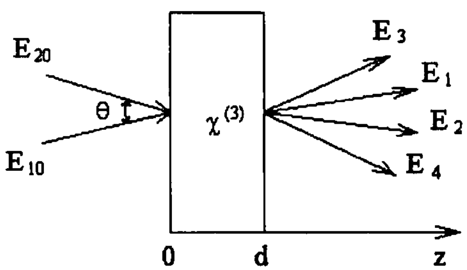


FIGURE A.1 Raman-Nath diffraction of light along the coordinate z in a medium with a cubic nonlinearity $\chi^{(3)}$ (here $\mathbf{E}_{10}$, $\mathbf{E}_{20}$ are the fields incident on the sample; $\mathbf{E}_3$ is the field diffracted from $\mathbf{E}_{10}$; $\mathbf{E}_4$ is the field diffracted from $\mathbf{E}_{20}$; $\mathbf{E}_{1,2}$ are the fields at the output of the medium; and Θ is the scattering angle).

For our discussion, we choose frequency-degenerate Raman-Nath scattering (diffraction) by a fine spatial grating subjected to a laser field as the nonlinear optical process used to generate the twin photon fields. This process can be implemented with particular effectiveness, e.g., in nematic liquid crystals (NLC), whose large cubic nonlinearity $\chi^{(3)}$ allows us to induce an effective grating in the optical field of a cw laser[18,19] and thereby make accurate observations of the quantum properties of the optical fields.

Figure A.1 shows a sketch of the process we will investigate. Amplitudes $\mathbf{E}_{10}$ and $\mathbf{E}_{20}$ (wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) correspond to optical fields at the input to the sample ($z = 0$), propagating at a certain angle Θ to one another. As a result of the nonlinear interaction, waves $\mathbf{E}_{10}$ and $\mathbf{E}_{20}$ write a grating in the medium, by which they are themselves diffracted (we denote the two original waves at the output of the medium by $\mathbf{E}_{1,2}$). This leads to multiple diffraction of the light by the spatially periodic structure. Higher diffraction orders (i.e., optical fields at the output with wave vectors $\mathbf{k}_j$, j>2) undergo interference extinction in this process, except in the case of fine phase gratings. The theoretical analysis of this problem is relatively simple; conditions for fineness of a grating in the case of nematic liquid crystals are determinated, e.g., in Ref.17.

Furthermore, since the amplitude of a diffracted wave decreases with increasing order, in the simplest case it is sufficent to limit ourselves to first-order diffraction, i.e., to assume, for example, that the incident wave $\mathbf{E}_{10}$ at the output is split into two waves - a transmitted wave $\mathbf{E}_1$ and a scattered (diffracted) wave $\mathbf{E}_3$. Wave $\mathbf{E}_{20}$ is likewise divided into waves $\mathbf{E}_2$ and $\mathbf{E}_4$ (Ref.18). In the language of nonlinear optics, this process is "four-wave mixing", on a component of the nonlinear polarization of the light in the medium, $\mathbf{P}(\mathbf{E}_{3,4}) = \chi^{(3)}\mathbf{E}_1\mathbf{E}_2\mathbf{E}^*_{1,2}$.

In this case, the operator for the optical field has the form

$$\mathbf{E} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\sum_{j=1}^{4}\left(\mathbf{l}_j a_j \exp(i\mathbf{k}_j\mathbf{r} - i\omega t) - \mathrm{H.c.}\right) \tag{A.1}$$

where t is the time, $\mathbf{r}$ the spatial coordinate, ω the optical carrier frequency, $\mathbf{l}_j$ the polarization vector (in what follows we will consider the scalar case), $\mathbf{V}$ the volume, ε_0 the dielectric constant, and a_j an annihilation operator (for brevity we do not write the operator sign) for the j-th wave (j=1-4, Fig.A.1).

For the general type of scattering under discussion here, the wave vectors for the j-th mode, $\mathbf{k}_j$ (where $|\mathbf{k}_1| = |\mathbf{k}_2| = |\mathbf{k}_3| = |\mathbf{k}_4| \equiv k$), satisfy the following phase matching conditions:

$$2\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3 + \Delta\mathbf{k}', \qquad 2\mathbf{k}_2 = \mathbf{k}_1 + \mathbf{k}_4 + \Delta\mathbf{k}, \tag{A.2}$$

where $\Delta\mathbf{k}$, $\Delta\mathbf{k}'$ are the phase detunings. In what follows we will assume the angles $\Theta_{i,j}$ between the corresponding wave vectors are small ($\Theta_{13} \approx \Theta_{12} \approx \Theta_{24} \equiv \Theta$). In this case we can set $|\Delta\mathbf{k}'| = |\Delta\mathbf{k}| \approx k\Theta^2/2$ for waves propagating near the direction of the z axis. We note that in our problem we cannot, in general, neglect the phase detunings $\Delta\mathbf{k}$ and $\Delta\mathbf{k}'$, because there is a spatial distribution of modes (in contrast to four-wave mixing or parametric amplification, where either the condition of phase matching is fulfilled automatically or it is assumed that this detuning is insignificant).

We define the Fano factor as the corresponding mean-square dispersion normalized by the level of fully coherent quantum fluctuations:

$$F_\pm = \frac{\left\langle(\Delta N_\pm)^2\right\rangle}{\langle N_\pm\rangle} = \frac{1}{\langle n_2\rangle + \langle n_3\rangle} \times \left[\left\langle(\Delta n_2)^2\right\rangle + \left\langle(\Delta n_3)^2\right\rangle \pm \left(\langle\Delta n_2 \Delta n_3\rangle + \langle\Delta n_3 \Delta n_2\rangle\right)\right], \tag{A.3}$$

where $N_{\pm} = a_2^+ a_2 \pm a_3^+ a_3$. For coherent light $F_{\pm} = 1$. Quantum fluctuations are suppressed for those quantum states of the field for which the modes are correlated ($\langle a_i^+ a_i a_j^+ a_j \rangle > \langle n_i \rangle \langle n_j \rangle$) and anticorrelated ($\langle a_i^+ a_i a_j^+ a_j \rangle < \langle n_i \rangle \langle n_j \rangle$) for the quantities F_- and F_+ respectively (where i,j=2,3).

Our analysis shows that the generation of correlations between modes and suppression of the Fano factor below the level of full coherence for Raman-Nath diffraction in a cubic nonlinear medium (NLC) are there possible with high efficiency. Quantum correlations of the individual mode intensities - the transmitted modes $a_2(z)$ and $a_3(z)$ [which scatters from $a_{10}(z)$] - make it possible for the Fano factor to decrease below the fully-coherent noise level for the sum and difference photon numbers.

The quasiclassical approximation we have used in our calculations has important implications from a practical point of view. This is due to the fact that the detectors used in schemes for measuring these quantum effects (see, e.g., Ref. 14) respond to the average numbers of photons, $\langle n_f(z) \rangle = n_{jc}(z) + n_{js}(z)$, at the output of the medium, where these photon numbers should be amplified considerably. For some real parameter of NLC we have shown that a considerable suppresion of quantum fluctuations is possible (see Figs.A.2, A.3). However, if the intensity of the probe field a_2 is sufficient for photodetection of $\langle n_j(z) \rangle$, then once we set $a_{30} = 0$ at the input of the medium, numerical calculations indicate that a decrease in the Fano factor F_- below the level of full coherence of 90% is achievable.

The scheme we have proposed has a number of advantages compared to analogous existing schemes used to observe nonclassical optics effects. For the case of liquid crystals these phenomena are especially efficient. The numerical parameters used in our paper correspond to nematic liquid crystals, whose use

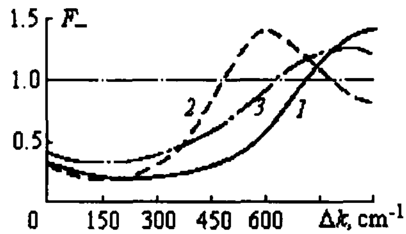


FIGURE A.2 Computed dependence of F_ on the phase detuning Δk . Here and in what follows, the value F_=1 (the horizontal dotted-dashed line) corresponds to the level of full coherence.

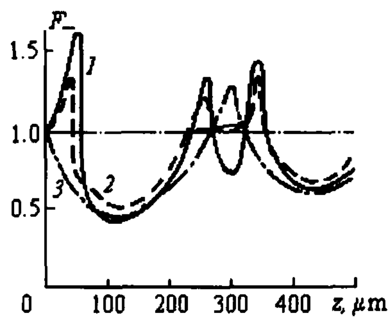


FIGURE A.3 Computed dependence of F_ on interaction length z.

provides the unique opportunity to observe optical effects in the fields of fairly-low-power cw lasers with high coherence properties. In these materials a large gain can be ahieved due to the high orientational nonlinearity of the medium ($\chi^{(3)} \approx n_2 \approx 2.26 \times 10^{-4}$ cm^2/W, input pump intensity $I_{10} \approx 1-10$ W/cm^2 ; Ref.17). Furthermore, mode selectivity with respect to scattering angle Θ also provides a number of other advantages in the experiment. First of all, Θ and therefore Δk as well can be varied once we have chosen the region of largest suppression of the Fano factor, i.e., the parametric limit. Second, for this case the modes $a_{2,3}(z)$ at the output of the medium are already spatially separated and do not require further conversion, e.g., in a polarization light divider used in some schemes for recording quantum fluctuations of the biphoton field (compare with Ref.14).

Acknowledgement

This work was supported by the Russian Found of Basic Research.

References

1. A.P. Alodjants, S.M. Arakelian, *Laser Physics* (special issue of "Nonclassical Light"), **4**, 765 (1994).
2. A.P. Alodjants, S.M. Arakelian, *Laser Physics*, **5**, 812 (1995); *Zh. Eksp. i Teor. Fiz.*, **107**, 1792 (1995) (*JETP*, **80**, 995 (1995)).
3. A. Ekert in *Proc. of the 14th International Conference on Atomic Physics*, edited by D. Wineland et al (AIP Press, NewYork, 1995), p.450.
4. T. Pellizzari, S.A. Gardiner, J.I. Cirac, P.Zoller, *Phys. Rev. Lett.*, **75**, 3788 (1995).
5. A.V. Krasheninnikov, L.A. Openov, *Pis'ma ZhETP*, **64**, 214 (1996) (*Sov. JETP Letters*); ibid S.N. Molotkov, p.219.
6. B. Daino, G. Grgory, S. Wabnitz, *J.Appl.Phys.*, **53**, 4512 (1985) S.F. Feldman, D.A. Weinberger, H.G. Winful, *J.Opt.Soc.Am B*, **10**, 1191 (1993).
7. A.A. Mayer, *Russian Uspekhi Fiz. Nauk*, **165**, 1037 (1995); ibid **166**, 1171 (1996).
8. A.P. Alodjants, A.Yu. Leksin, S.M. Arakelian, *Izvestia RAN, Ser. Fizicheskaya*, **60**, 46 (1996)

9. W.A. Shurkliff, *Polarized light* (Harvard University Press, Cambridge, Massachusetts, 1962).

10. A.P. Alodjants, S.M. Arakelian, A.S. Chirkin, *Quantum and Semiclassical Optics, part B*, **9** (1997) - in press.

11. L. Solymar, *Superconductive tunneling and applications* (Chapman and Hall Ltd., London, 1972).

12. H.M. Gibbs, *Optical bistability: controlling light with light* (Academic Press Inc., 1985).

13. A.P. Alodjants, S.M. Arakelian, *"Quantum nondemolition measurements of the phases and polarization Stokes parameters of optical fields", Zh. Eksp. i Teor. Fiz.(JETP, 1997)* - in press

14. A. Heidmann, R.J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Gamy, *Phys. Rev. Lett.*, **59**, 2555 (1987); C.Fabre, E. Giacobino, A. Heidmann, L. Lugiato *et al.*, *Quant. Opt.*, **2**, 159 (1990).

15. R.S. Bondurant, P. Kumar, J.H. Shapiro, and M.D. Levenson, *J. Opt. Soc. Am. B*, **1**, 390 (1984).

16. A.P. Alodjants, S.M. Arakelian, and Yu.S. Chilingarian, *Izv. Akad. Nauk SSSR, Ser. Fiz.*, **55**, 357 (1991); *Opt. Spektrosk.*, **71**, 635 (1991) [*Opt. Spectrosc. (USSR)*, **71**, 365 (1991)].

17. S.M. Arakelian, and Yu.S. Chilingarian, *Nonlinear Optics of Liquid Crystals* [in Russian] (Nauka, Moscow, 1984).

18. I.C. Khoo and T.H. Lin, *Phys. Rev. A*, **39**, 4036 (1989).

19. S.D. Durbin, S.M. Arakelian, and Y.R. Shen, *Opt. Lett.*, **7**, 145 (1982).

20. A.N. Chaba, J.J. Collet, and D.F. Walls, *Quant. Opt.*, **4**, 19 (1992).